March 21, 2014

International Baccalaureate®
Baccalauréat International
Bachillerato Internacional

Mathematics

Standard level

Specimen papers 1 and 2

SPEC/5/MATME/SP1/ENG/TZ0/XX

International Baccalaureate®
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MATHEMATICS
STANDARD LEVEL
PAPER 1

SPECIMEN

1 hour 30 minutes

Candidate session number

0	0							

Examination code

X	X	X	X	–	X	X	X	X

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all questions in the boxes provided.
- Section B: answer all questions on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- A clean copy of the ***Mathematics SL*** *formula booklet* is required for this paper.
- The maximum mark for this examination paper is *[90 marks]*.

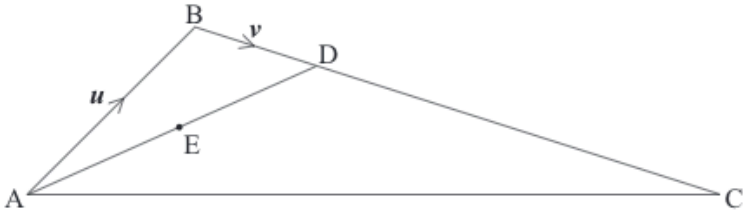
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A (44 Marks)

*Answer **all** questions in the boxes provided. Working may be continued below the lines if necessary.*

1. *[Maximum mark: 7]*

In the following diagram, $\boldsymbol{u} = \vec{AB}$ and $\boldsymbol{v} = \vec{BD}$.



The midpoint of $\vec{AD}$ is E and $\frac{BD}{DC} = \frac{1}{3}$.

Express each of the following vectors in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$.

(a) $\vec{AE}$ *[3 marks]*

(b) $\vec{EC}$ *[4 marks]*

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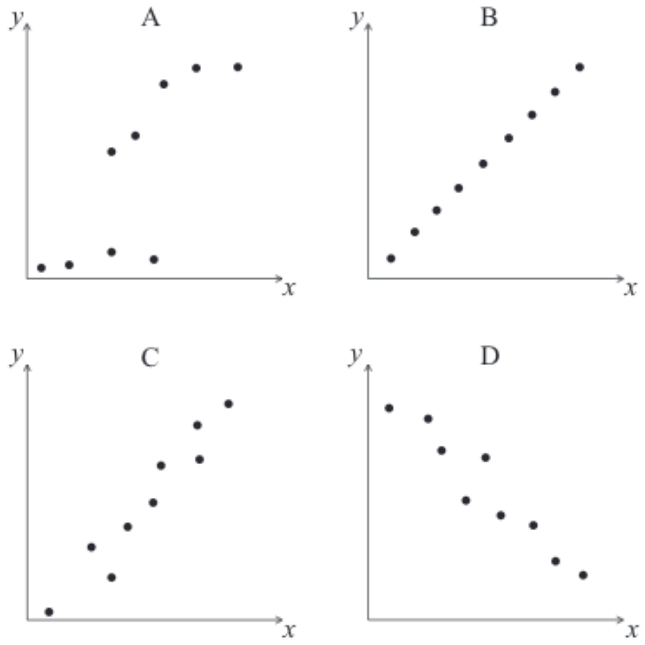
2. *[Maximum mark: 5]*

There are nine books on a shelf. For each book, x is the number of pages, and y is the selling price in pounds (£). Let r be the correlation coefficient.

(a) Write down the possible minimum and maximum values of r. *[2 marks]*

(b) Given that $r = 0.95$, which of the following diagrams best represents the data. *[1 mark]*

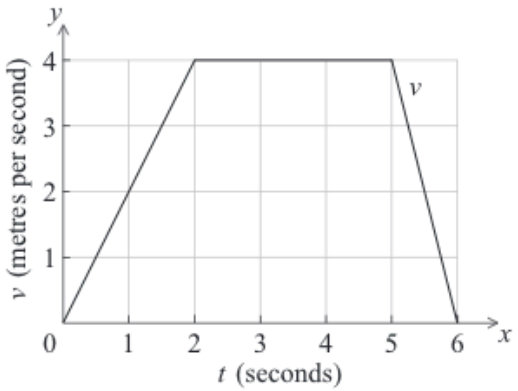


(c) For the data in diagram D, which **two** of the following expressions describe the correlation between x and y?

perfect, zero, linear, strong positive, strong negative, weak positive, weak negative *[2 marks]*

3. *[Maximum mark: 6]*

A toy car travels with velocity $v\ \text{ms}^{-1}$ for six seconds. This is shown in the graph below.



(a) Write down the car's velocity at $t = 3$. *[1 mark]*

(b) Find the car's acceleration at $t = 1.5$. *[2 marks]*

(c) Find the total distance travelled. *[3 marks]*

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4. *[Maximum mark: 5]*

A data set has a mean of 20 and a standard deviation of 6.

(a) Each value in the data set has 10 added to it. Write down the value of

(i) the new mean;

(ii) the new standard deviation. *[2 marks]*

(b) Each value in the original data set is multiplied by 10.

(i) Write down the value of the new mean.

(ii) Find the value of the new variance. *[3 marks]*

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5. *[Maximum mark: 7]*

(a) Find $\int \frac{e^x}{1+e^x} dx$. *[3 marks]*

(b) Find $\int \sin 3x \cos 3x \, dx$. *[4 marks]*

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6. *[Maximum mark: 7]*

The expression $6\sin x\cos x$ can be expressed in the form $a\sin bx$.

(a) Find the value of a and of b. *[3 marks]*

(b) Hence or otherwise, solve the equation $6\sin x\cos x = \frac{3}{2}$, for $\frac{\pi}{4} \leqslant x \leqslant \frac{\pi}{2}$. *[4 marks]*

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7. *[Maximum mark: 7]*

Given that $f(x) = \frac{1}{x}$, answer the following.

(a) Find the first four derivatives of $f(x)$. *[4 marks]*

(b) Write an expression for $f^{(n)}(x)$ in terms of x and n. *[3 marks]*

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*Do **NOT** write solutions on this page.*

SECTION B (46 Marks)

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 15]*

Let $f(x) = 3(x+1)^2 - 12$.

(a) Show that $f(x) = 3x^2 + 6x - 9$. *[2 marks]*

(b) For the graph of f

(i) write down the coordinates of the vertex;

(ii) write down the y-intercept;

(iii) find both x-intercepts. *[7 marks]*

(c) **Hence** sketch the graph of f. *[3 marks]*

(d) Let $g(x) = x^2$. The graph of f may be obtained from the graph of g by the following two transformations

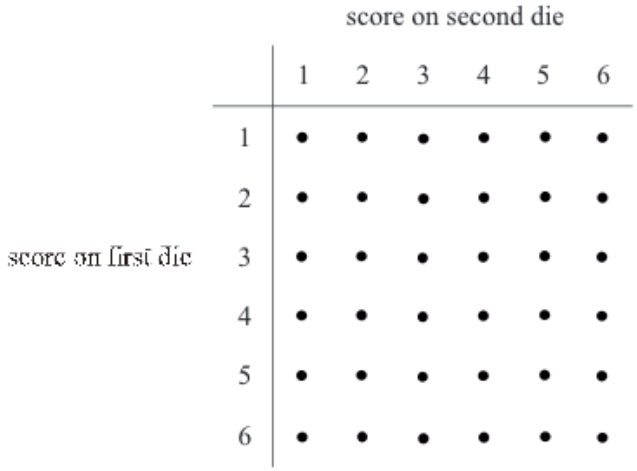
a stretch of scale factor t in the y-direction,

followed by a translation of $\begin{pmatrix} p \\ q \end{pmatrix}$.

Write down $\begin{pmatrix} p \\ q \end{pmatrix}$ and the value of t. *[3 marks]*

9. *[Maximum mark: 14]*

Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.



Let X be the sum of the scores on the two dice.

(a) (i) Find $\mathrm{P}(X = 6)$.

(ii) Find $\mathrm{P}(X > 6)$.

(iii) Find $\mathrm{P}(X = 7 \mid X > 6)$. *[6 marks]*

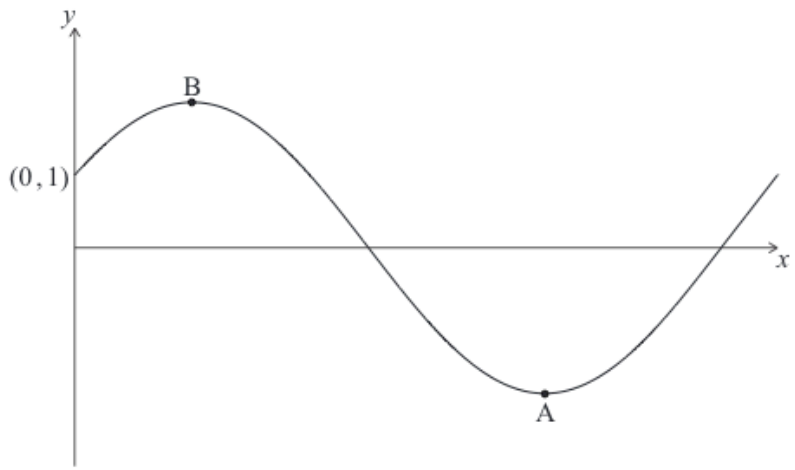
(b) Elena plays a game where she tosses two dice.

If the sum is 6, she **wins** 3 points.
If the sum is greater than 6, she **wins** 1 point.
If the sum is less than 6, she **loses** k points.

Find the value of k for which the game is fair. *[8 marks]*

10. *[Maximum mark: 17]*

Let $f(x) = \cos x + \sqrt{3}\sin x$, $0 \leq x \leq 2\pi$. The following diagram shows the graph of f.



The y-intercept is at $(0, 1)$, there is a minimum point at A (p, q) and a maximum point at B.

(a) Find $f'(x)$. *[2 marks]*

(b) Hence

(i) show that $q = -2$;

(ii) verify that A is a minimum point. *[10 marks]*

(c) Find the maximum value of $f(x)$. *[3 marks]*

The function $f(x)$ can be written in the form $r\cos(x - a)$.

(d) Write down the value of r and of a. *[2 marks]*